A Slice of Legal Cybernetics: LPV Modelling and Control of Speeding Fines

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***Abstract*—The main purpose of this paper is to provide a different approach to speed limit enforcement, more precisely, to speeding fines. The goal is to construct such a speeding fine system that in result can minimise speeding and consequently decrease road traffic accidents, traffic jams, and exhaust emissions. In order to reduce speeding, a truly effective speed limit enforcement is required. Therefore, we propose making speeding fines dependent on offenders' average monthly income after tax and on feedbacks such as the number of speed limit exceedances (both individually and globally in a particular country). The introduced feedbacks open new possibilities for implementing cybernetic approaches such as model-based closed-loop controller design techniques. In this paper a possible way of modelling and control is presented based on the linear parameter-varying (LPV) paradigm, Lyapunov theory of stability, and guaranteed cost. The introduced time-delay LPV model is developed and tuned based on data from the *Statistical Office of Slovak Republic* (SOSR) and on data obtained by detailed questionnaire survey. Then a static output-feedback gain-scheduled controller is designed, which is validated on a long-term simulation based on actual and predicted data from the SOSR. The main contribution of this paper is to provide a possible way of modelling and control of speeding fines using modern control and cybernetic techniques.**

***Index Terms*—Speeding fines, legislation, modelling, linear parameter-varying systems, robust control, gain scheduling, guaranteed cost, legal cybernetics.**

I. Introduction

According to Global status report on road safety published last year by the World Health Organisation, over 1.2 million people die each year on the world's roads, with millions more sustaining serious injuries and living with long-term adverse health consequences. Globally, road traffic crashes are a leading cause of death among young people, and the main cause of death among those aged 15-29 years [1]. Speeding (i.e. exceeding the speed limit stated in a particular road area) is one of the most critical risk factors for road traffic injuries and a contributory factor in ca. 30% of all road-related deaths [2]. Furthermore, compliance with speed limits is a highly important factor in terms of traffic flow control as well [3], [4]. In light of the foregoing, speeding has direct effects on road safety and traffic jams as well as it indirectly impacts exhaust emissions and fuel consumption.

Road safety management, including statement of speed limits and speeding sanctions primarily belongs to the area of legal regulation. Enforcement of speed limits is essential in order to make them truly effective. Indeed, where countries have changed their national speed limits but have taken little supporting action to enforce them, there have been very limited benefits [1]. Traffic rules are stated on both national and international level in such legal sources as international treaties, regulations, directives, national acts, ordinances, etc. Violation of these legally binding rules results in immediate creation of a secondary legal (liability) relationship [5], which includes sanctioning. Depending on the countries' legal systems, there is a wide scale of speeding sanctions: imposing fines, suspension or confiscation of the driving licence for a certain period of time, even imprisonment. One can however fairly ask, whether these sanctions really can dissuade drivers from speeding. In fact, legal regulation itself is not sufficiently effective, especially in consideration of frequent law changes and too delayed feedback, which may result in further law changes, forming thus a vicious circle. The main shortcoming is that because of the delayed feedback the law change is also delayed (sometimes too late) and the desired correction may become impracticable. That is why developing a method which can eliminate these obstacles and thus make the legal regulation more effective, is of crucial importance. Our research focuses on regulation of speeding fines using a cybernetic approach.

The idea of applying cybernetic (or control theory) methods in social science is not new. One can find several cybernetic applications in almost every branches of social science: in economics [6]–[8], psychology [9]–[11], sociology [12]–[14], education [15], [16], law [17]–[19], as well as in multidisciplinary context [20], [21]. Our paper further highlights the interdisciplinary nature of cybernetics and its great utilisation in the area of legal regulation.

Our approach can be used in intelligent traffic systems that control traffic flows by dynamic speed limits taking into account the weather conditions as well. Thus, intelligent traffic systems are able to reduce traffic jams and minimise road accidents (for instance, see the Swedish Vision Zero initiative [22]). For effective functioning of intelligent traffic systems, speed limit enforcement is of crucial importance. That is exactly what our ideas presented in this paper are intended to improve. The main goal is to construct a speeding fine system which dissuades drivers from speeding and thus guarantees as high road safety as possible.

In this paper a possible way of modelling and control of speeding habits is presented using linear parameter-varying (LPV) techniques. The notion of LPV systems was introduced first by Shamma, J.S. in 1988 to model gain scheduling [23]. Today the LPV paradigm has become a standard formalism in the area of systems and controls with lot of contributions

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devoted to analysis, controller and filter design, and system identification of these models [24]. For a more comprehensive survey of the field, readers are also referred to survey papers [25]–[27] and references therein.

The rest of the paper is organized into six sections. The introduction is followed by preliminaries and problem formulation in *Section II*. The measuring instruments and data obtained are described in *Section III*, which is followed by the LPV modelling in *Section IV*. The robust discrete-time gain-scheduled controller design is presented in *Section V*, and the proposed methodology is applied and validated on long-term simulation in *Section VI*. Concluding remarks close the paper in *Section VII*.

The mathematical notations of the paper are as follows. $D \in \mathbb{R}^{m \times n}$ denotes the set of real $m \times n$ matrices. Given a symmetric matrix $P = P^T \in \mathbb{R}^{n \times n}$, the inequality $P > 0$ ($P \geq 0$) denotes the positive definiteness (semi-definiteness) of the matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions. I denotes the identity matrix of corresponding dimensions. Notation for interval of numbers between a and b including endpoints a and b is $\langle a, b\rangle = \{x \in \mathbb{R} | a \leq x \leq b\}$. Furthermore, $\theta \in \langle \underline{\theta}, \overline{\theta}\rangle \in \Omega$ denotes that $\theta \in \Omega$ belongs to the bounded set $\underline{\theta} \leq \theta \leq \overline{\theta}$.

II. Preliminaries and problem formulation

The main goal of this paper is to propose an innovative method for stating speeding fines, using a linear parameter-varying (LPV) modelling- and control technique. The suggested approach may make the sentencing optimal, dynamic, and depending on certain variables such as number of speedings (globally and individually) and average monthly income. According to our idea, there is no need for legislative change in case of fine amount changes; moreover, any "manual" fine modifications are not necessary either. The only requirement is to enact a control law and replace the actual statements about speeding fines. In this paper, we present a possible approach to define the control law.

A. Contextual boundaries

Although there is a wide range of obligations pertaining to road traffic participants as well as a broad scale of sanctions in case of their unlawful violation, the proposed approach focuses only on fines imminent for speeding. Extensions are addressed to further studies. The legislative background and statistical data used in this paper stem from Slovak Republic (hereafter only SR).

B. Legal regulation of speeding fines in SR

Currently, there is a dual system governing the speeding punishment procedure. Act on Road Traffic [28] contains graded, exact speeding fines depending on the extent of speeding (14 fine levels with ceiling of 798 EUR). Fines according to this act are imposed to the vehicle keeper based on objective liability (i.e. the vehicle keeper is liable for the detected speeding regardless of fault). The system based on objective liability is used in case of automatic law enforcement, i.e. during exceeding the speed limit the vehicle's licence plate is recorded and the vehicle keeper receives a speeding fine via a post. Act on Minor Offences [29] however, builds on subjective liability where fault is required and thus fines are imposed to that particular person who has violated the speed limit. It belongs to non-automatic law-enforcement. In this case, immediately after speeding detection, a police officer stops the offender and imposes a speeding fine or other sentence (e.g. confiscation of driving licence) on the spot. If the offender appeals against the speeding fine imposed on the spot, the competent police authority begins an administrative procedure, where speeding fines are higher. Currently, there are three fine grades, both for speeding inside built-up areas and outside them. The upper fine limit is 1000 EUR which is extended by confiscation of driving licence for a certain period of time (from 6 months to 3 years). In terms of fining on the spot, the maximal speeding fine is 800 EUR. Despite in the last seven years the number of road accidents has been reduced quite significantly, speeding is still the most frequent violation of road traffic rules in SR [30]. Furthermore, same speeding fines apply to all people regardless of their average monthly income or the number of speedings during a particular period of time. In our opinion, these variables should also be taken into account when imposing speeding fines. Globally, the actual amount of speeding fine should be stated in percentage of the particular person's actual average monthly income after tax and should also depend on number of global and individual speedings in a certain period of time (e.g. during the last 365 days).

C. Assumptions

There are some requirements which have to be met in order to make our proposed fining system effective and truly functional. Firstly, speed limits have to be justifiable and adaptive to various conditions (weather, road, actual traffic). A number of countries apply dynamic speed limits on their motorways and in the near future, dynamic speed limit systems will spread to other road types as well. For purposes of this paper, we assume that this requirement is perfectly fulfilled. The second requirement is rather an issue of legal relevancy. Because the proposed fine structure is significantly determined by the average monthly income after tax, we assume that the legal system can eliminate such speculations (law evasions) which result in that the official income does not reflect the reality.

III. Measuring instruments and data obtained

Our research is based on two data sources: the Statistical Office of SR and data obtained by questionnaire survey. We decided to carry out a questionnaire survey in order to test our preliminary hypotheses (like men in average rather tend to violate speed limits than women, as well as younger people are more likely to exceed speed limits than elders). In addition, we wanted to know, how much percent of their average monthly income after tax people do already not want to sacrifice as a speeding fine. The questionnaire focused on drivers living in SR. According to the Slovak Road Traffic Act currently in force, people can obtain a driving licence for vehicle categories A1 and B1 from the age of 16 years. The samples were distributed via the Internet and were filled out by 478 Slovak Internet users. The number of respondents was almost equal in terms of genders (50.1% women and 49.9% men) and roughly comparable in light of different age groups (see Fig. 1).

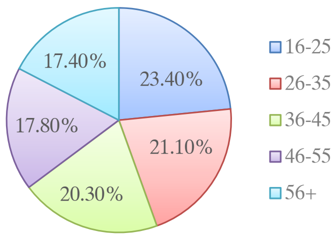

Fig. 1. Proportion of respondents based on age groups.

Within the survey we examined the frequency of speedings inside built-up areas and outside them during the last 12 months, according to genders and different age categories. The respondents could choose a number from a frequency scale (0-10) where 0 meant that the particular person did not exceed speed limits at all during the last 12 months and 10 was equivalent to every time exceedance. Figure 2 shows the frequency of speedings between women and men inside built-up areas. As it is reflected by the figure, ca. 16% of men tended to comply with speed limits inside built-up areas. In case of women, this number is 4% higher.

Equally 5.5% of both men and women violated speed limits during the last 12 months more than usually (ca. with 62% frequency). Roughly 2% of women exceeded speed limits inside built-up areas each time when driving. This number is 17% higher in case of male drivers. According to measured data, inside built-up areas women were more likely to comply with speed limits and did not exceed them too frequently. In case of men, number of those who violated speed limits was increasing from 65% speeding frequency (more than usually) to 100% (always) by ca. 4%.

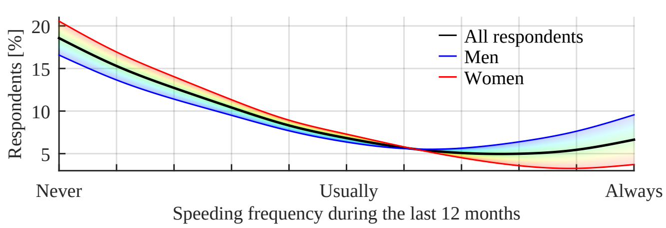

Fig. 2. Frequency of speedings *inside* built-up areas (genders)

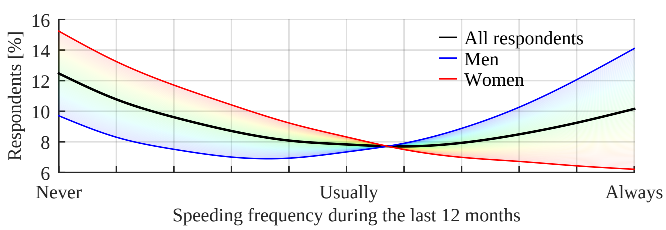

Fig. 3. Frequency of speedings *outside* built-up areas (genders)

Figure 3 reflects gender-based speeding frequency outside built-up areas. In this regard, during the last 12 months, almost 10% of man complied with speed limits each time when driving (6% less than inside built-up areas). This number is ca. 5% higher in case of female drivers (but inside built-up areas 5% more women adhered to speed limits each time when driving). Roughly 8% of both men and women exceeded speed limits a bit more then usually (with 57% speeding frequency). Ca. 6% of women violated speed limits each time when driving during the last 12 months (4% more than inside built-up areas), while in case of male drivers, 14% drove always above the speed limit (which is 8% higher than in case of women and 4% more than inside built-up areas).

Figure 4 shows frequency of speedings in terms of different age groups. The columns indicate age categories inside and outside built-up areas, while rows refer to frequency of speedings (from 0 - never to 10 - always, 5 means usually). As shown in the figure, people aged between 16-25 and 26-35 are more likely to exceed speed limits both inside and outside built-up areas. Frequency of speedings is inversely proportional to age. In average, there is a 1.02 frequency unit difference between speeding inside and outside built-up areas (from "more than sometimes" to "a bit more than usually").

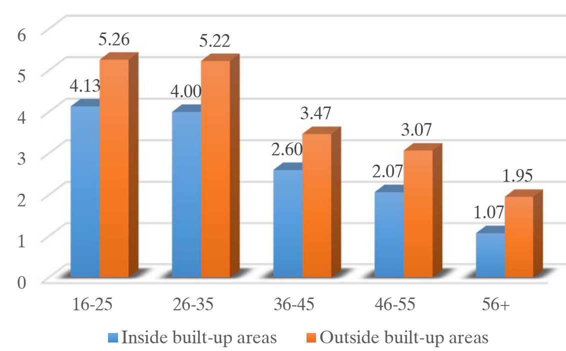

Fig. 4. Frequency of speedings inside and outside built-up areas (age groups)

Figure 5 shows the dependency of respondent's speeding on the amount of imminent fine (which is stated as a percentage of their average monthly income after tax). The vertical axis refers to number of respondents (in %) who would sacrifice a certain amount of their average monthly income after tax (AMIat) as a speeding fine. The horizontal axis indicates the percentage of AMIat in a logarithmic scale. The figure shows that ca. 97% of respondents would not be dissuaded from speeding in case if the speeding fine were 1% of their AMIat. If the fine were set to 10% of AMIat, there would be only ca. 49% of respondents who would continue in exceeding speed limits (approximately half as much as in terms of 1%). However, in case of speeding fine stated to 100% of AMIat, only 1.76% of respondents would be willing to sacrifice this amount as a punishment. In other words, ca. 98% of respondents would comply with speed limits knowing that their whole AMIat is imminent as a speeding fine.

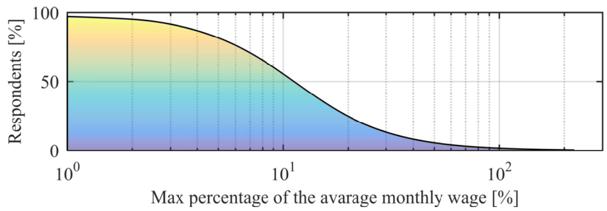

Fig. 5. Dependence of speeding on the amount of speeding fine counted from the average monthly income after tax

TABLE I
NUMBER OF DETECTED SPEEDINGS

Year	Number of detected speedings	Number of used radars (operating hours per day)
2015	305218	360 (24), 60 (8), 40 (3.5)
2014	255754	290 (24), 60 (8), 40 (3.5)

TABLE II
NUMBER OF CARS WITH VALID REGISTRATION CERTIFICATE IN SLOVAK REPUBLIC DURING 2011-2015

Year	2011	2012	2013	2014	2015
Amount	2442231	2537976	2622939	2725538	2843809

IV. I/O TIME-DELAY LPV MODEL

In our opinion, the speeding fine shall not be an exact sum of money. It should rather express a percentage, i.e. a certain percent of the offender's average monthly income after tax. It would ensure more equitable conditions for people with different income amounts because the fine would cause equal rate of financial loss for everyone. Regarding the fine calculation structure, the value of the final individualised speeding fine (u_f) should depend on the following variables:

- y (See (15)), where ξ denotes the i-th day's speeding detections and γ marks the weighted number of traffic enforcement cameras used on the i-th day.
- extent of exceedance, i.e. by how much km/h the particular offender exceeded the speed limit ($\alpha_f \in \langle 0, 70\rangle$ $[km/h]$),
- frequency of speedings, i.e. how many times the offender exceeded the speed limit during the last 365 days ($\beta_f \in \langle 0, 10\rangle$),
- speeding locality, i.e. whether the offender violated the speed limit inside ($\sigma = 1$) or outside ($\sigma = 0$) a built-up area.

In light of the foregoing, we define the equation of the speeding fine as follows:

$$u_f = u + f_1(\alpha_f, \sigma) + f_2(\beta) \tag{1}$$

where

$$\begin{aligned} f_1(\alpha_f, \sigma) &= f_{10}(\alpha_f)\sigma + f_{11}(\alpha_f)(1-\sigma), \\ f_2(\beta_f) &= 0.0011\beta_f^5 - 0.0173\beta_f^4 + 0.1437\beta_f^3 \\ &\quad -0.4064\beta_f^2 + 1.009\beta_f - 0.735, \\ f_{10}(\alpha_f) &= 9\times 10^{-09}\alpha_f^5 - 2\times 10^{-06}\alpha_f^4 + 9\times 10^{-05}\alpha_f^3 \\ &\quad -0.0021\alpha_f^2 + 0.0282\alpha_f - 0.1123, \\ f_{11}(\alpha_f) &= 6\times 10^{-09}\alpha_f^5 - 1\times 10^{-06}\alpha_f^4 + 7\times 10^{-05}\alpha_f^3 \\ &\quad -0.0015\alpha_f^2 + 0.0186\alpha_f - 0.0688, \end{aligned}$$

furthermore, u is the gain-scheduled output feedback controller's output (see (24)). The $f_1(\alpha_f, \sigma) \in \langle 0, 154.59\%\rangle$ function indicates the fine raise depending on the exceedance extent. To determine the exact rate of fine raise, we based on a fine gradation enacted in the Act on Road Traffic which is currently in force in SR. Then we changed the stated sums of money to a percentage value. This value has been calculated from the average of the minimal income and average income after tax currently applicable in SR. The $f_2(\beta_f) \in \langle 0, 50\%\rangle$ function is intended to tighten the speeding fine imposed for recidivists. The function value is exponentially proportional to the number of individual relapses during the last 365 days. There is an upper limit: the tightening must not be more than 50%. In addition, we defined speeding fine thresholds as follows:

$$u_{f_{min}} \leq u_f \leq u_{f_{max}}, \tag{2}$$

where $u_{f_{min}} = 2.96\%$ of the offender's average monthly income after tax or 15 EUR (depending on which amount is higher), and $u_{f_{max}} = 300\%$ of the offender's average monthly income after tax or 10 000 EUR (depending on which amount is lower).

Based on the survey results and data from the Statistical Office of SR, the following model has been obtained, for input $u(t)$ (i.e. basal value for speeding fines) and output $\bar{y}(t)$ (i.e. number of daily detections fractioned by the used weighted number of speedcams - day per hour):

$$\begin{aligned} \dot{\bar{x}}(t) &= -1/T_f(\mu,\omega)\bar{x}(t) + \frac{K_f(\bar{y},\mu,rc)}{T_f(\mu,\omega)}u(t), \\ \bar{y}(t) &= \bar{x}(t) \end{aligned} \tag{3}$$

where the time constant has been approximated as:

$$T_f(\mu,\omega) = \left(\frac{15}{\mu} + 15\right)\omega, \tag{4}$$

and wherein $\omega \in \mathbb{R} \in \langle \underline{\omega}, \overline{\omega}\rangle$ is the media factor, and $\mu \in \mathbb{R} \in \langle \underline{\mu}, \overline{\mu}\rangle$ is the actual amount of speed radars compared to the amount of radars used in 2015 (i.e. 460). The gain K_f, based on the static input-output characteristic obtained from the survey (Fig. 5), and data from the measurements has been approximated as:

$$K_f(\bar{y},\mu,rc) = \frac{y}{fitmodel\left(\frac{-\left(y - \frac{y_0 y_m}{100}\right)100}{\frac{y_m}{\mu}}\right)}, \tag{5}$$

where $y_0 = 99.0279$, $y_m = 2.3622 - rc$, furthermore

$$\begin{aligned} fitmodel(y_e) &= (c_1 + c_2cos(y_ec_0) + c_3sin(y_ec_0) \\ &+c_4cos(2y_ec_0) + c_5sin(2y_ec_0) + c_6cos(3y_ec_0) \\ &+c_7sin(3y_ec_0) + c_8cos(4y_ec_0) + c_9sin(4y_ec_0) \\ &+c_{10}cos(5y_ec_0) + c_{11}sin(5y_ec_0) + c_{12}cos(6y_ec_0) \\ &+c_{13}sin(6y_ec_0) + c_{14}cos(7y_ec_0) + c_{15}sin(7y_ec_0)) \end{aligned}$$

wherein $c_0 = 0.009623$, $c_1 = 4.879 \times 10^{10}$, $c_2 = -7.498 \times 10^{10}$, $c_3 = -4.144\times10^{10}$, $c_4 = 3.07\times10^{10}$, $c_5 = 4.884\times10^{10}$, $c_6 = -1.658 \times 10^9$, $c_7 = -2.929 \times 10^{10}$, $c_8 = -4.731 \times 10^9$, $c_9 = 9.846 \times 10^9$, $c_{10} = 2.294 \times 10^9$, $c_{11} = -1.633 \times 10^9$, $c_{12} = -4.467 \times 10^8$, $c_{13} = 5.137 \times 10^7$, $c_{14} = 3.106 \times 10^7$, and $c_{15} = 1.274 \times 10^7$.

The coefficient rc can be calculated as:

$$rc = \left(1 - \frac{rc_b}{rc_a}\right) rc_f, \tag{6}$$

where rc_a is the actual number of cars with a valid registration certificate in SR, rc_b is the number of cars with a valid registration certificate in SR 365 days prior to the current day, and rc_f is an influencing factor, which has been determined as $rc_f = 0.1$, based on the statistical data from the last 5 years (Tab. 2).

The nonlinear model (3) can be transformed to the following discrete LPV model using the forward Euler discretization approach with sample time $T_s = 1$ [31]:

$$\begin{aligned} \bar{x}(k+1) &= \bar{A}_x(\theta(k))\bar{x}(k) + \bar{B}_x(\theta(k))u(k), \\ \bar{y}(k) &= \bar{C}_x\bar{x}(k), \end{aligned} \tag{7}$$

where

$$\bar{A}_x(\theta(k)) = 1 + a_0 + a_1\theta_1,\ \bar{B}_x(\theta) = b_0 + b_2\theta_2,\ \bar{C}_x = 1, \tag{8}$$

furthermore,

$$\theta_1 = \frac{-1/Tf(\mu,\omega) - a_0}{a_1}, \tag{9}$$

$$\theta_2 = \frac{K_f(\bar{y},\mu,rc)/Tf(\mu,\omega) - b_0}{b_1} \tag{10}$$

The coefficients a_0, a_1, b_0 and b_1 are calculated so as to maintain the $\theta_{1,2}(k) \in \langle -1, 1\rangle$:

$$a_0 = \frac{max(-1/Tf(\mu,\omega)) + min(-1/Tf(\mu,\omega))}{2}, \tag{11}$$

$$a_1 = \frac{max(-1/Tf(\mu,\omega)) - min(-1/Tf(\mu,\omega))}{2}, \tag{12}$$

$$b_0 = \frac{max(\frac{K_f(\bar{y},\mu,rc)}{Tf(\mu,\omega)}) + min(\frac{K_f(\bar{y},\mu,rc)}{Tf(\mu,\omega)})}{2}, \tag{13}$$

$$b_1 = \frac{max(\frac{K_f(\bar{y},\mu,rc)}{Tf(\mu,\omega)}) - min(\frac{K_f(\bar{y},\mu,rc)}{Tf(\mu,\omega)})}{2}. \tag{14}$$

In our case, for $\bar{y} \in \langle 0.0781, ym\rangle$, $\mu \in \langle 0.5, 6\rangle$, $rc \in \langle -0.01, 0.01\rangle$, and $\omega \in \langle 0.5, 1\rangle$ coefficients are $a_0 = -0.0683$, $a_1 = 0.0460$, $b_0 = 0.0123$ and $b_1 = 0.0114$.

Based on the data from the Statistical Office of SR, we know that the actual state $\bar{x}(k)$ is affected by disturbances (i.e. higher during Christmas/Easter or other holidays, festivals, and during spring and summer). For this reason, as the system output, the following variable has been chosen:

$$y = \frac{\sum_{i=-365}^{1}\left(\frac{\gamma_i}{\xi_i}\right)}{365}, \tag{15}$$

where γ_i is the i-th day's speeding detections and ξ_i is the weighted number of speed radars used on the i-th day (Tab. 1). The weighting is based on operating hours of speed radars per day.

In light of the foregoing, the system (7) can be extended to the following time-delay LPV system:

$$\begin{aligned} x(k+1) &= A_x(\theta(k))x(k) + B_x(\theta(k))u(k) \\ &\quad + A_dx(k-\tau), \\ y(k) &= C_xx(k), \end{aligned} \tag{16}$$

where, in our case $\tau = 365$, $x(k) = [\bar{x}(k),\ x_2(k)]^T$ and

$$\begin{aligned} A_x(\theta(k)) &= \begin{bmatrix} \bar{A}_x(\theta(k)), & 0 \\ 1/365, & 1 \end{bmatrix}, \\ Ad &= \begin{bmatrix} 0, & 0 \\ -1/365, & 0 \end{bmatrix}, \\ B_x(\theta) &= \left[\bar{B}_x(\theta(k)),\ 0\right]^T,\ C_x = [0,\ 1]. \end{aligned} \tag{17}$$

V. Robust discrete GS controller design

A. Theoretical prelude

We were looking for an optimal controller design method, which can ensure the closed-loop system stability and in addition minimise the cost to go. Therefore, as a basis, we decided to use one of our previous controller design approaches. More precisely, the gain-scheduled controller design approach presented in [32], which guarantees the closed-loop stability and guaranteed cost for a prescribed rate of change of scheduled parameters for continuous-time affine LPV systems. A robust version of this approach can be found in [33] for affine LPV systems with polytopic model uncertainty. The discrete-time version of this approach can be found in [34] for quadratic stability as well as for affine quadratic stability.

Because of the time-delay in the LPV system (16), the mentioned approach (based on quadratic stability) is extended for time-delay LPV systems in a new and interesting way.

Consider a time-delay LPV system in the form (16). This system can be transformed to an equivalent LPV system as follows:

$$\begin{aligned} x(k+1) &= A(\theta(k))x(k) + B(\theta(k))u(k), \\ y(k) &= Cx(k), \end{aligned} \tag{18}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, and $y(k) \in \mathbb{R}^l$ are the state, control input, and the measured output vectors, respectively. The matrix functions $A(\theta(k)) \in \mathbb{R}^{n\times n}$ and $B(\theta(k)) \in \mathbb{R}^{n\times m}$ are assumed to depend on the scheduling variable $\theta(k) \in \langle\underline{\theta}, \overline{\theta}\rangle \in \Omega$. This variable $\theta(k) = [\alpha_1, \ldots, \alpha_{N_p}, \beta_1, \ldots, \beta_{Nu}, \lambda_1, \ldots, \lambda_n]$ can be split to a part where it is assumed that the scheduling parameters $\alpha_i(k)$ $i = 1, 2, \ldots, N_p$ are constant or time varying and can be measured or estimated (therefore used in the controller), and to a part, where the scheduling parameters $\beta_i(k)$, $i = 1, 2, \ldots, N_u$ are constant or time varying but unknown (uncertain) parameters, and to a part where the parameters $\lambda_i(k)$, $i = 1, \ldots, n$ are the calculated parameters to cover the time-delay states.

$$L(\theta(k)) = L_0 + \sum_{i=1}^{N_p} L_i\alpha_i(k) + \sum_{i=1}^{N_u} L_{N_p+i}\beta_i(k) + \sum_{i=1}^{n} L_{N_p+N_u+i}\lambda_i(k) = L_0 + \sum_{i=i}^{p} L_i\theta_i(k) \quad (19)$$

with $L(\theta(k)) = \{A(\theta(k)),\ B(\theta(k))\}$. In addition A_0, B_0, A_i, B_i, $i = 1, 2, \ldots, p$, and C are constant matrices with appropriate dimensions. The scheduled parameters for time-delay states are calculated as follows:

$$\lambda_i(k) = \left(\frac{\frac{x_i(k-\tau)}{x_i(k)} - z_{i_0}}{z_{i_1}}\right),\ x_i(k) \neq 0,\ i = 1, \ldots, n. \quad (20)$$

The coefficients z_{i_0} and z_{i_1} are calculated so as to maintain the $\lambda_i(k) \in \langle -1, 1\rangle$:

$$z_{i_0} = \frac{max\left(\frac{x_i(k-\tau)}{x_i(k)}\right) + min\left(\frac{x_i(k-\tau)}{x_i(k)}\right)}{2}, \quad (21)$$

$$z_{i_1} = \frac{max\left(\frac{x_i(k-\tau)}{x_i(k)}\right) - min\left(\frac{x_i(k-\tau)}{x_i(k)}\right)}{2}. \quad (22)$$

In our case, the time-delay system (16) can be transformed as follows (with only one $\lambda(k)$ for $x_1(k-\tau)$):

$$\begin{gathered} A_0 = \begin{bmatrix} \bar{A}_{x_0}, & 0 \\ \frac{1-z_0}{365}, & 1 \end{bmatrix}, A_1 = \begin{bmatrix} \bar{A}_{x_1}, & 0 \\ 0, & 0 \end{bmatrix}, \\ A_2 = \begin{bmatrix} \bar{A}_{x_2}, & 0 \\ 0, & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0, & 0 \\ \frac{-z_1}{365}, & 0 \end{bmatrix}, \\ B_i = \bar{B}_{x_i}, i = 0, \ldots, 2;\ B_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, C = C_x. \end{gathered} \quad (23)$$

The output feedback gain-scheduled control law is considered for discrete-time PSD controller in the form:

$$u(k) = \Big(K_P(\theta(k))e(k) + K_S(\theta(k))\sum_{i=0}^{k} e(i) + K_D(\theta(k))(e(k) - e(k-1))\Big), \quad (24)$$

where $e(k) = y(k) - w(k)$ is the control error, $w(k)$ is the reference signal, and matrices $K_P(\theta(k))$, $K_S(\theta(k))$, $K_D(\theta(k))$ are controller gain matrices in the form (19) with $L(\theta(k)) = \{K_P(\theta(k)),\ K_S(\theta(k)),\ K_D(\theta(k))\}$ (for SISO systems they are scalars). Note that the number of controller gain matrices is only N_p, the rest $N_u + n$ are equal to zero.

With the assumption that the reference signal $w(k)$ is bounded, the control law for $w(k) = 0$ can be rewritten as follows:

$$u(k) = \Big(K_P(\theta(k))y(k) + K_S(\theta(k))\sum_{i=0}^{k} y(i) + K_D(\theta(k))(y(k) - y(k-1))\Big), \quad (25)$$

We can extend the system with two state variables [35]:

$$z_1(k) = \textstyle\sum_{i=0}^{k-2} y(i), \quad z_2(k) = \sum_{i=0}^{k-1} y(i), \quad (26)$$

furthermore, substituting expressions $y(k-1) = z_2(k) - z_1(k)$ and $\sum_{i=1}^{k} y(k) = z_2(k) + y(k)$ to the control law (25), one can obtain:

$$u(k) = \Big(\left(K_P(\theta(k)) + K_S(\theta(k)) + K_D(\theta(k))\right) y(k) + K_S(\theta(k))z_2(k) - K_D(\theta(k))(z_2(k) - z_1(k))\Big). \quad (27)$$

The control law (27) can be transformed to the following state space matrix form:

$$u(t) = F(\theta(k))\tilde{y}(k), \quad (28)$$

where $\tilde{y} = [y(k),\ z_1(k),\ z_2(k)]^T$ is the extended measured output vector and

$$F(\theta(k))^T = \begin{bmatrix} K_P(\theta(k)) + K_S(\theta(k)) + K_D(\theta(k)) \\ K_D(\theta(k)) \\ K_S(\theta(k)) - K_D(\theta(k)) \end{bmatrix}.$$

Substituting the control law (28) to the system (18), the following closed-loop system is obtained:

$$\tilde{x}(k+1) = A_{cl}(\theta(k))\tilde{x}(k), \quad (29)$$

where $\tilde{x}(k) = [\ x(k),\quad z_1(k),\quad z_2(k)\ ]^T$ and

$$\begin{gathered} A_{cl}(\theta(k)) = A_r(\theta(k)) + B_r(\theta(k))F(\theta(k))C_r, \\ A_r(\theta(k)) = \begin{bmatrix} A(\theta(k)), & 0, & 0 \\ 0, & 0, & I \\ C, & 0, & I \end{bmatrix}, C_r = \begin{bmatrix} C, & 0, & 0 \\ 0, & I, & 0 \\ 0, & 0, & I \end{bmatrix}, \\ B_r(\theta(k)) = [\ B(\theta(k)),\quad 0,\quad 0\ ]^T. \end{gathered}$$

To assess performance quality, with possibility to obtain different performance quality in each working point, a quadratic cost function described in our previous paper [36] is used:

$$J_{df}(\theta(k)) = \sum_{k=0}^{\infty} J_d(\theta(k)) \quad (30)$$

where

$$\begin{gathered} J_d(\theta(k)) = \tilde{x}(k)^T Q(\theta(k))\, \tilde{x}(k) + u(k)^T R\, u(k), \\ Q(\theta(k)) = Q_0 + \sum_{i=1}^{p} Q_i\theta_i(k),\ Q_i = Q_i^T \geq 0,\ R > 0, \end{gathered}$$

furthermore, Q_0, $Q_i \in \mathbb{R}^{(n+2l)\times(n+2l)}$, $R \in \mathbb{R}^{m\times m}$ are symmetric positive definite (semidefinite) and definite matrices, respectively.

B. Robust gain-scheduled PSD controller design

The robust gain-scheduled PSD controller design is based on the following lemmas:

Lemma 1: Consider the closed-loop system (29). Closed-loop system (29) is quadratically/affinely quadratically stable with guaranteed cost if the following inequality holds:

$$B_e(\theta(k)) = \max_u \{\Delta V(\theta(k)) + J_d(\theta(k))\} \leq 0, \quad (31)$$

for $\forall\theta(k) \in \Omega$ and $\Delta\theta(k) \in \Omega_t$. For proof see [37].

Lemma 2: Consider a scalar quadratic function of $\theta \in \mathbb{R}^p$.

$$f(\theta_1, \ldots, \theta_p) = a_0 + \sum_{i=1}^{p} a_i\theta_i + \sum_{i=1}^{p}\sum_{j>i}^{p} b_{ij}\theta_i\theta_j + \sum_{i=1}^{p} c_i\theta_i^2,$$

and assume that $f(\theta_1, \ldots, \theta_p)$ is multi-convex, that is $\frac{\partial^2 f(\theta)}{\partial \theta_i^2} = 2c_i \geq 0$ for $i = 1, 2, \ldots, p$. Then $f(\theta)$ is negative for all $\theta \in \Omega$ if and only if it takes negative values at the corners of θ. [38]

Using *Lemmas 1* and *2* the following theorem can be obtained:

Theorem 1: Closed-loop system (29) is robust quadratically stable with guaranteed cost if, for a given weighting matrices $Q_i, R, i = 0, 1, 2, \ldots, p$ there exist:

- positive definite matrix $P > 0$,
- positive semi-definite symmetric matrices $G_i \geq 0$, $i = 1, \ldots, p$,
- controller matrices K_{P_i}, K_{S_i}, K_{D_i}, $i = 0, 1, \ldots, p$,

such that the following inequalities holds at the corners of the scheduled parameters θ_i, $i = 1, \ldots, p$:

$$\begin{aligned} M(\theta(k), \lambda(k)) &= M_0 + \sum_{i=1}^{p} M_i \theta_i(k) \\ &+ \sum_{i=1}^{p} \sum_{j>i}^{p} M_{ij} \theta_i(k) + \sum_{i=1}^{p} M_{ii} \theta_i^2(k) \leq 0, \end{aligned} \tag{32}$$

$$M_{ii} \geq 0; \quad i = 1, 2, \ldots, p, \tag{33}$$

where $M_{ii} = M_{ij} + G_i$, $i = j$, and

$$M_0 = \begin{bmatrix} -P + Q_0, & C_r^T F_0^T, & A_{cl_0}^T \\ F_0 C, & -R^{-1}, & 0 \\ A_{cl_0}, & 0, & P_x \end{bmatrix}$$

$$M_i = \begin{bmatrix} Q_i, & C_r^T F_i^T, & A_{cl_i}^T \\ F_i C_r, & 0, & 0 \\ A_{cl_i}, & 0, & 0 \end{bmatrix},$$

$$M_{ij} = \begin{bmatrix} 0, & 0, & A_{cl_{ij}}^T \\ 0, & 0, & 0 \\ A_{cl_{ij}}, & 0, & 0 \end{bmatrix},$$

$$P_x = X^{-1}(P - X)X^{-1} - X^{-1},$$

$$A_{cl0} = A_{r0} + B_{r0} F_0 C_r, \; A_{clij} = B_{ri} F_j C + B_{rj} F_i C_r,$$

$$A_{cli} = A_{ri} + B_{ri} F_0 C_r + B_{r0} F_i C_r, \; A_{clii} = B_{ri} F_j C_r.$$

Proof 1: Proof is based on *Lemmas 1* and *2*. Assume that the Lyapunov function is in the form:

$$V(k) = \tilde{x}(k)^T P \tilde{x}(k). \tag{34}$$

The first difference of the Lyapunov function is then:

$$\begin{aligned} \Delta V(\theta(k), \lambda(k)) &= \tilde{x}(k)^T H_{\Delta V} \tilde{x}(k) < 0 \\ H_{\Delta V} &= A_{cl}(\theta(k))^T P A_{cl}(\theta(k)) - P \end{aligned} \tag{35}$$

Substituting the obtained first difference of the Lyapunov function (35) and the cost function (30) to the Bellman-Lyapunov function (31) we can obtain:

$$\begin{aligned} W(\theta(k)) &= A_{cl}(\theta(k))^T P A_{cl}(\theta(k)) \\ &- P + Q(\theta(k)) + C_r^T F(\theta(k))^T R F(\theta(k)) C_r \leq 0. \end{aligned} \tag{36}$$

Using Schur complement we can obtain:

$$W(\theta(k)) = \begin{bmatrix} W_{11}, & W_{21}^T, & W_{31}^T \\ W_{21}, & W_{22}, & W_{32}^T \\ W_{31}, & W_{32}, & W_{33} \end{bmatrix} \leq 0, \tag{37}$$

where

$$\begin{aligned} W_{11} &= -P + Q(\theta(k)), & W_{22} &= -R^{-1}, \\ W_{21} &= F(\theta(k)) C_r, & W_{32} &= 0, \\ W_{31} &= A_{cl}(\theta(k)), & W_{33} &= -P^{-1}. \end{aligned}$$

One can linearise the nonlinear part as follows:

$$lin(-P^{-1}) \leq X^{-1}(P - X)X^{-1} - X^{-1}, \tag{38}$$

where in each iteration holds $X_i = P_{i-1}$ (i – actual iteration step).

After we extend (37) to affine form we can obtain (32) and (33) by applying *Lemma 1* and relaxing the multi-convexity requirement as follows:

$$M(\theta(k)) = W(\theta(k)) + \textstyle\sum_{i=1}^{p} G_i \theta_i^2 \geq W(\theta(k)) \tag{39}$$

where $G_i \geq 0$, $i = 1, \ldots, p$ are symmetric semi-definite auxiliary matrices.

The obtained controller parameters, for weighting matrices $Q_0 = I_n$, $Q_{1,2} = 0$, $R = I_m$, are as follows:

$$K_P(\theta(k)) = -5.2745 + 1.9443\theta_1 + 0.1888\theta_2 \tag{40}$$

$$K_S(\theta(k)) = -0.0362 + 0.0185\theta_1 + 0.0018\theta_2 \tag{41}$$

$$K_D(\theta(k)) = -0.1148 + 0.0252\theta_1 - 0.0117\theta_2 \tag{42}$$

VI. Simulation results

The simulations were made based on fictive policy measures of a fictive country (called 'Utopialand') in a 22-year perspective.

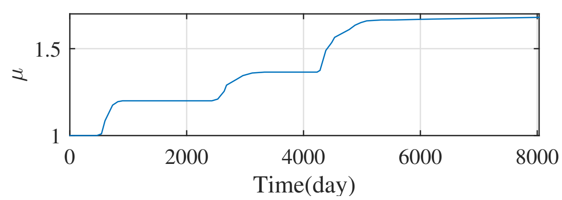

Fig. 6. Simulation results - μ

Fig. 6 indicates how the number of speed radars operating on roads is changing (μ). The actual amount of speed radars greatly influences the number of speeding detections and the drivers' speeding habits, respectively. Therefore, it is definitely important to take into account when designing the controller. As it is shown on the figure, during the second year the number of speed radars increased by 20% because police offices in bigger cities had received new speed measuring devices. Since this action had effectively helped to reduce the number of speedings, between the 7-th and 9-th year other cities and villages became enriched by new speed measuring instruments. Finally, as a result of a large investment between the 11-th and 14-th year, a speed radar system (that has actually been already successfully used in the Netherlands) was set up. Longer highway sections were equipped with fixed speed radars measuring the average speed of vehicles passing through these roads. Thus, by the 22^{nd} year the number of radars had been increased by almost 70%.

Fig. 7 denotes the change of the media factor ω. It can influence people's reaction time to speeding fine change. In

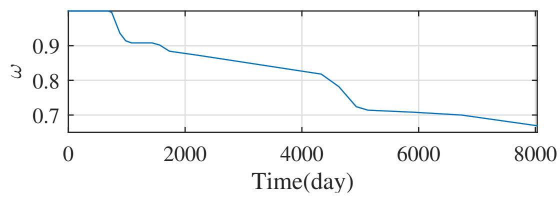

Fig. 7. Simulation results - ω

Utopialand, the new speeding fine system was mediated by introducing on television at the beginning of the 3^{rd} year. Then an online forum had been created informing about the actual speeding fines. During the following years the Internet became enriched with lots of new users who used the forum actively, including older people as well. During the 13-th year, a new mobile app became commercialised which was intended to inform drivers about the current amount of speeding fines.

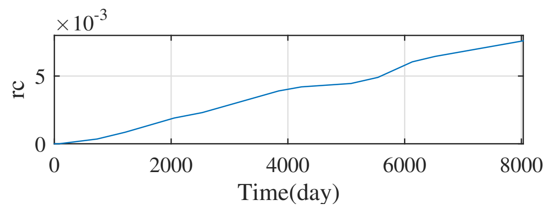

Fig. 8. Simulation results - rc

Fig. 8 shows the increasing number of cars with a valid registration certificate (these are real data based on the Slovak Ministry of Interior's statistics). The number of cars is presented by a coefficient (6).

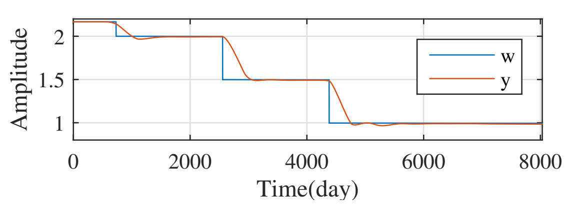

Fig. 9. Simulation results - w, y

Fig. 9 indicates the change in the number of average daily speeding detections per radar. w is the reference value which can vary depending on the government policy. Our simulation assumptions are the following: after Utopialand had increased the number of active speed radars by 20%, governing bodies aimed to reduce the number of average daily speeding detections per radar by 0.2. In the 5^{th} year, the European Union enacted a directive according to which every country was obliged to take measures that should lead to a 25% decrease in the number of speedings. During the 11^{th} year, Utopialand introduced a new program (that time already successfully functioning in Sweden) called 'Vision Zero', which had been designed to reduce the number of traffic accidents to zero and consequently to significantly reduce the number of speedings. y denotes the measured output. Overshoots were caused by the effects of factors presented on figures 6, 7 and 8.

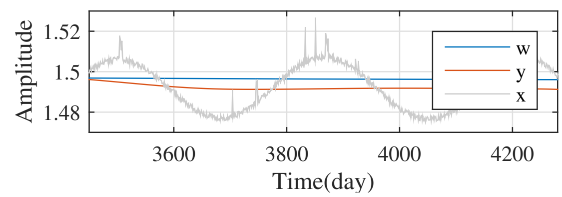

Fig. 10. Simulation results - w, y, and x

Fig. 10 is a zoomed part of the fig. 9 (2 years), which contains (in addition to the reference value w and the system state x) the actual number of average daily speeding detections per radar (the average is counted from detections per radar in the last 365 days - y). In the simulation, the disturbances and noises are based on real statistical data provided by the Presidium of the Police Force in SR. The peeks indicate different cases that increased the number of speedings (Christmas holidays in the 10^{th} year followed by days of enhanced speed measurement, then there were Easter holidays, and finally the summer festival and tourist season when lots of foreign cars passed through the country).

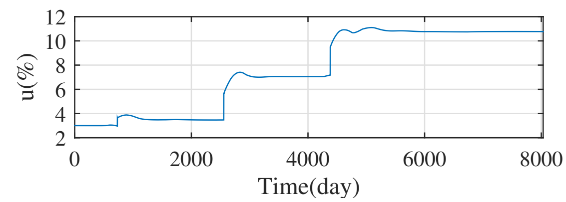

Fig. 11. Simulation results - u

On fig. 11, one can see how the gain-scheduled output feedback controller's output is changing.

VII. Conclusion

The main contribution of the paper, in addition to theoretical novelties is that it is a feasible application of control theory in law, a real and practical example of legal cybernetics. Closed-loop control of speeding fines is a unique and yet not applied method of legal regulation. The idea of making speeding fines dependent on the offender's average monthly income after tax can also be considered as a novelty. It would ensure more equitable conditions for people with different income amounts because the fine would cause equal rate of financial loss for everyone. We suggest speeding fines to depend on the number of global and individual speedings in a certain period of time as well. Our main objective was to construct such a speeding fine system that in result can minimise speeding and consequently decrease road traffic accidents, traffic jams, and exhaust emissions. The proposed approach thus can be successfully utilised in traffic flow control, and can play a significant role in strengthening and increasing traffic safety. As for future works, we are planning to collect statistical as well as personal (questionnaire-based) data in Sweden and initiate cooperation with the Swedish Traffic Agency.

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